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LETTERS TO THE EDITOR

VIBRATION CONTROL FOR A FLEXIBLE TRANSMISSION SHAFT WITH AN AXIALLY SLIDING SUPPORT

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1. INTRODUCTION

With modern trends in engineering, machine components are designed to be as light as possible to minimize both the material and the operating costs. In addition, these components are driven at high speeds to satisfy the fast production demands. Hence, it is not uncommon to have a power transmission shaft operating above the first few critical speeds of its structure. Although such a shaft may operate safely in the steady state, damaging transient resonance can occur during acceleration to the super-critical operating speed. A simple active control technique has been demonstrated earlier to avoid transient resonance by varying the critical frequencies, as the shaft accelerates through them [1]. This technique was investigated further in the authors' previous work [2].

Previous work concentrated on the use of a midspan support which could be either present (activated) or not present (de-activated). Presence of the midspan additional support, of course, would stiffen the flexible shaft and shift its critical frequencies higher. By having a choice of two different frequency response functions, a control strategy could be established. The frequency response function was switched instantaneously (by activating and de-activating the midspan support) to select the smaller response at any given frequency of excitation. Hence, the possibility of the traversing speed of the transmission shaft to coincide with a critical speed could be avoided. However, this switching support could cause large vibrations when it clamped the shaft during activation. In addition, the best location for a switching support was usually different for each critical frequency to be passed during acceleration. The switching support could be placed at only one location. This limitation results in a compromise to attenuate vibrations at more than one critical speed. The best compromise location for a switching support allowed acceleration to $7{\cdot}5\omega_1$, between ω_2 and ω_3, where ω_i refers to the ith critical frequency. A sliding support is proposed here to move to a different location to bypass each critical speed and make acceleration to beyond $40\omega_1(\sim\omega_5)$ possible. The suggested configuration is shown in Figure 1.

2.1. *Frequency response*

This study deals with the acceleration of a simplified shaft model of length, $L = 1{\cdot}65$ m, stiffness, $EI = 65$ Nm^2, and mass per unit length, $\rho A = 0{\cdot}576$ kg/m, as shown in Figure 1. A 0·50 kg mass (representing a power transmission component such as a gear with a comparable mass to that of the shaft's) with imbalance $m\varepsilon = 0{\cdot}001$ kgm is located at the point $L_M/L = 0{\cdot}60$. This shaft was modelled with ten Euler beam [3] elements with a numerical simulation program in FORTRAN. The boundary conditions due to permanent and sliding supports are assumed to be simple support. The finite elements approximation

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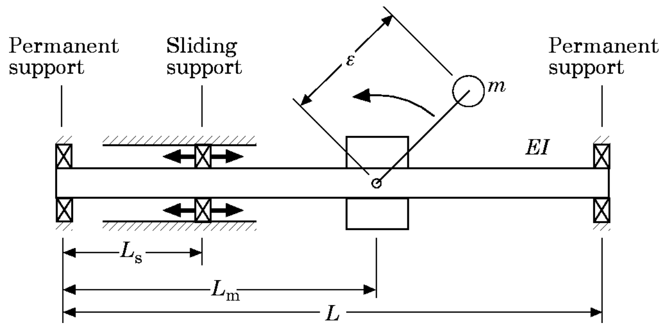

Figure 1. Schematic of the flexible beam with a sliding bearing.

yielded five natural frequencies to within 3% of the example cases compiled in reference [4]. Solution of the eigenvalue problem was then used to produce the dynamic response of the flexible beam by the standard modal superposition technique.

Figure 2(a) shows the frequency response of the displacement of the simply supported flexible beam subject to excitation of "$m\varepsilon\omega^2 \sin(\omega t)$" at the point $L_M/L = 0{\cdot}60$. The new displacement responses with an additional bearing in three different positions at $L_S/L = 0{\cdot}15(\triangle)$, $L_s/L = 0{\cdot}35$ ($\diamond$) and $L_S/L = 0{\cdot}55$ ($+$) are shown in Figure 2(b). If, for any given excitation speed, the support location with the minimum displacement response can be chosen, the effective displacement response will be the minimum of the three functions, shown by shading. This effective response is significantly smaller compared to any of the individual displacement responses, so the shaft with the sliding bearing should be capable of acceleration from zero speed to $40\omega_1$, just below ω_5, with effectively no resonance. This assertion will be verified next by simulating the transient response of the flexible beam model.

It should be noted in Figure 2(b) that this shaded area has been picked up to deliberately avoid an excessive number of actuations between different support locations. Otherwise, a smaller response could be indicated in this figure, as in the case of around 150 rad/s. Starting with the $L_S/L = 0{\cdot}55$ ($+$), switching over to $L_S/L = 0{\cdot}15$ ($\triangle$) around 120 rad/s, and switching back to $L_S/L = 0{\cdot}55$ around 135 rad/s, would enable catching the trough of the $L_S/L = 0{\cdot}55$ case. However, the price of this marginally smaller response would be two additional actuations.

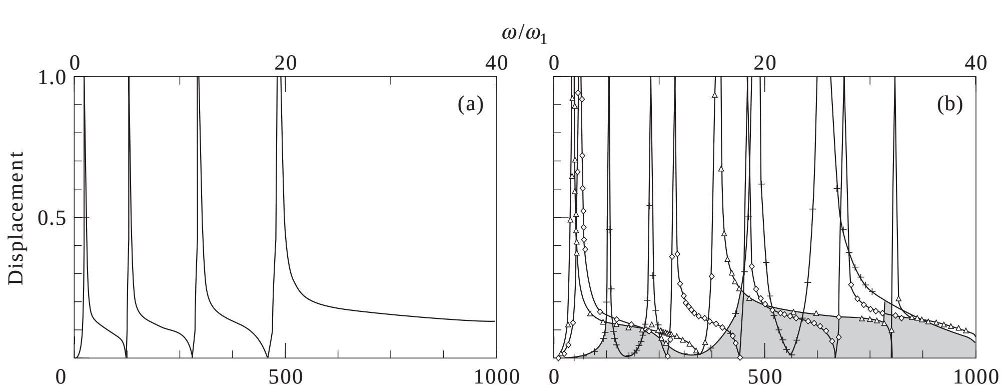

Figure 2. Displacement response for the (a) simply supported shaft and (b) shaft with an added sliding bearing at $L_S/L = 0.15$ ($\triangle$), $0{\cdot}35$ ($\diamond$) and $0{\cdot}55$ ($+$).

2.2. *Transient solution*

The displacement spectra in Figure 2 are the steady state response of the system. Power transmission shafts, on the other hand, are made out of common metals with very light inherent damping. Thus, for any accelerating shaft, the transient vibrations will likely play a significant role. For this reason, a computer program was prepared to numerically predict the transient response of the system during acceleration. Cases were analysed by assuming that the angular speed of the excitation, ω, followed a constant slope (acceleration) from rest to the operating speed. The numerical predictions were obtained, assuming the excitation speed was constant over each time step, by standard mode superposition. 5000 time steps were used from rest to the operating speed. This number was found to be large enough for convergence with ample safety margin.

In general, the slower a shaft accelerates through a resonance, the more evident the resonance effects will be. For cases with swift acceleration to $40\omega_1$ over time less than $40T_1$ (T_1 represents the fundamental period), the shaft did not resonate during acceleration. Rapid acceleration to avoid resonance is well known but not often feasible. The example case to demonstrate the axially sliding bearing technique is acceleration from rest to $40\omega_1$ over a time period $400T_1$, or 100 s.

Figure 3(a) shows the transverse vibration of the point $L_M/L = 0{\cdot}6$ on the uncontrolled system, a simply supported shaft, during acceleration from rest to $40\omega_1$. The peaks due

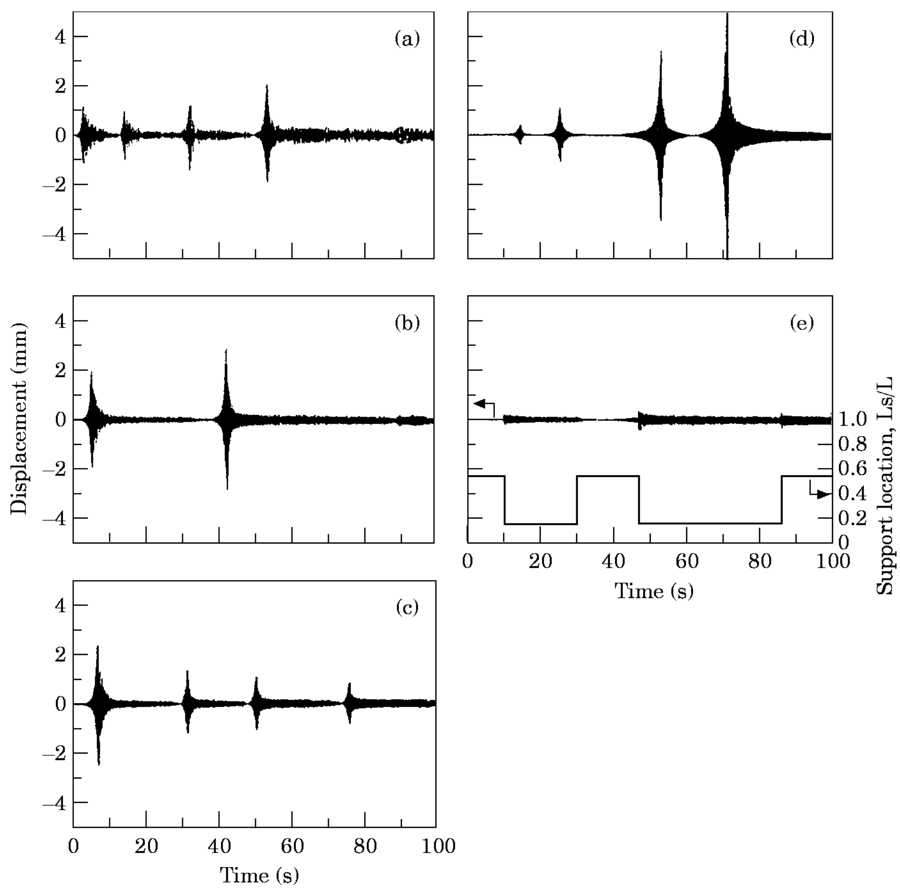

Figure 3. Transverse vibration history of the flexible beam at $L_M/L = 0.60$ with (a) no midspan support; stationary midspan support L_S/L at (b) 0·15, (c) 0·35, (d) 0·55 and (e) sliding support.

to resonance for the first four critical frequencies are apparent and the maximum amplitude of vibration is about 2 mm. Figures 3(b), 3(c) and 3(d) show the transverse vibration of the shaft at the point $L_M/L = 0{\cdot}6$ with a "stationary" additional support at the points $L_S/L = 0{\cdot}15$, $0{\cdot}35$, and $0{\cdot}55$, respectively. It can be seen that, for each case, the shaft has large amplitudes of vibration as it resonates around its critical speeds. The timing and the magnitude of the resonance response are different for each case since the shaft stiffness is a function of the support location. The maximum amplitude is 2·8 mm with $L_S/L = 0{\cdot}15$ and 5·2 mm with $L_S/L = 0{\cdot}55$. The best location for a stationary bearing is at the point $L_S/L = 0{\cdot}35$, which yields a maximum amplitude of about 2·5 mm, but the addition of a stationary support at all three locations actually increases the vibration instead of reducing it. Figure 3(e) shows the transverse vibration of the shaft with a support sliding "instantaneously" between the two discrete points $L_S/L = 0{\cdot}15$ and $L_S/L = 0{\cdot}55$. The location of the sliding bearing is $L_S/L = 0{\cdot}55$ for the ranges (0, 100) rad/s, (300, 464) rad/s, and (861, 1000) rad/s and $L_S/L = 0{\cdot}15$ for the rest of the time, shown as the staggered step function with its scale on the right. It can be seen in Figure 2(b) that the intermediate support location, $L_S/L = 0{\cdot}35$, is unnecessary since the displacement response for $L_S/L = 0{\cdot}35$ is never significantly lower than those of the $L_S/L = 0{\cdot}15$ and $L_S/L = 0{\cdot}55$.

The last actuation at 861 rad/s, could have been as early as 800 rad/s due to the third resonance of the case with $L_S/L = 0{\cdot}15$. However, this particular peak is quite sharp resulting in a very short duration over which the transient resonance takes place. In addition, there is always some delay between the peak response instant and the instant when the frequency of excitation matches the critical frequency. Hence, a delayed actuation could easily be tolerated.

For any shaft speed, the location for the sliding bearing is the location with minimum displacement response. Small jumps in the vibration amplitude can be seen at the times $t = 10$, $t = 46$ and $t = 86$ s. This occurs because the stiffness is rapidly decreased as the sliding support is moved instantaneously from $L_S/L = 0{\cdot}55$ to $L_S/L = 0{\cdot}15$. This can be explained by considering an analogous single-degree-of-freedom (SDOF) system with variable stiffness. If, during vibration, the spring of the SDOF was suddenly unstiffened, the mass would have to travel farther to store its kinetic energy as the potential energy in the spring. Thus a jump in vibration amplitude related to the kinetic energy would be expected. For this reason the switches in support location were activated when the shaft had low transverse velocity (and therefore low transverse kinetic energy). Since the instantaneous repositioning of the additional support is adopted here for computational ease, and since this repositioning would have to take place gradually in practice, the sudden jumps in Figure 3(e) are not considered to be a serious drawback of the suggested technique. It can be seen in Figure 3(e) that the vibration amplitudes with the active sliding support have been sharply reduced. Because the sliding support moved to minimize the steady state amplitude with respect to frequency, resonance was never encountered; as such, no resonance peaks are evident. The maximum amplitude of less than 0·5 mm represents a decrease of 75% from the initial simply supported case.

2.3. *Determining support locations*

The slider locations to produce the results presented in section 2.2 were determined by trial and error. A procedure is suggested here to reduce the number of trials, by examining the normalized mode shapes of the simply supported flexible beam.

Placing the additional support at a point with a large relative amplitude for the nth mode will have a significant stiffening effect on the nth critical speed. On the other hand, sliding the support to a point on the shaft where there is a node will have no effect on displacing

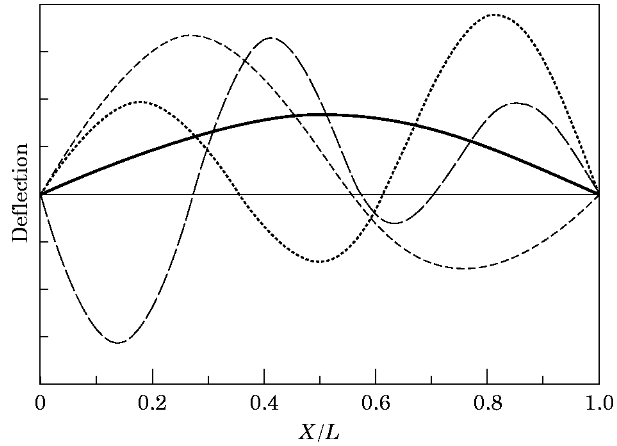

Figure 4. Mode shapes of the simply supported beam. X/L represents the axial distance from the left-hand-side permanent support: —, mode 1; ---, mode 2; . . . . , mode 3; — —, mode 4.

the corresponding critical frequency. Hence, referring to the first four mode shapes of the simply supported flexible beam shown in Figure 4, the addition of a support at $L_S/L = 0{\cdot}55$ has a much greater effect on the first mode than a support at $L_S/L = 0{\cdot}15$ or at $L_S/L = 0{\cdot}35$. This may be verified with the spectral information in Figure 2(b).

For resonance free operation around the nth unstiffened critical frequency, a good support location will be the one with considerable modal amplitude for the nth mode but relatively low modal amplitude for the preceding mode. It is desirable for the preceding mode to have low relative amplitude at the potential support location to minimize stiffening the preceding mode and raising the preceding critical frequency. If the preceding mode is stiffened too much, it may interfere and raise the displacement response around the nth critical frequency. Figure 4 demonstrates this as the second mode has relatively high amplitude near the point $L_S/L = 0{\cdot}35$. Examining Figure 2(b), it can be seen that the addition of a support at $L_S/L = 0{\cdot}35$ has stiffened the second natural frequency so that it is very close to the unstiffened third natural frequency.

3. CONCLUSIONS

It appears then that a sliding bearing can be successfully used to avoid resonance when accelerating to super-critical speeds, demonstrated past the first four critical frequencies to $40\omega_1$. Moving the sliding bearing smoothly should avoid the sudden amplitude jumps and impacts associated with the on/off type switching supports discussed earlier. Recommended future work includes the investigation of continuously variable bearing location, instead of discrete locations (instantaneous relocation of the slider bearing). Future work on optimising the slider bearing location versus frequency function would be welcome and experimental verification is also recommended to confirm the veracity of the computational results.

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